Monte Carlo Method for Optimizing Energy Distribution Networks

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Abstract

We describe a macroscopic mathematical framework and Monte Carlo algorithm for find- ing structures that best satisfy energy supply/demand constraints in a network. The method uses a Markov 'routing matrix' for directing flows in the algorithm's 'inner' loop while the elements of this matrix are periodically updated in an 'outer' loop using Monte Carlo sam- pling. A novel feature of the method is the way in which it 'evolves' network structures based upon minimizing an objective function we call the 'network roughness' function. We illustrate the efficacy of the method on an example problem.

Keywords: Monte Carlo, Simulated annealing, Energy distribution, Networks

1. Introduction and literature review

We describe research investigating a novel mathematical framework describing a macro- scopic network flow model in conjunction with a hybrid Monte Carlo-Markov matrix opti- mization algorithm for its solution. The overall aim of the optimization is to find network structures that best satisfy supply/demand constraints throughout the system. Investigating a novel mathematical framework describing a macroscopic network flow model in conjunc- tion with a hybrid Monte Carlo-Markov matrix optimization algorithm for its solution. The overall aim of the optimization is to find network structures that best satisfy supply/demand constraints throughout the system. The computational method is two-tiered. In the inside loop the probability elements of a Markov 'routing matrix' are found which are used to direct network flows. Their values are periodically updated in an 'outer' loop using Monte Carlo sampling in a manner reminiscent of simulated annealing. The overall aim of the method is to find network structures that best satisfy supply/demand constraints throughout the system.

Complex network behavior is important to many physical and chemical systems when interactions can be represented by network (graph) structures. These structures are mathe- matically represented by components (nodes) in the system that is linked together by edges. Examples of applications of these concepts include computer networks [1], biological systems [2–8], power supply grids [9–17] and financial risk management networks [18, 19]. The be- havior in these systems depends upon the pattern of interconnections between components in the network. Knowledge of this connectivity is key to answering many practical questions e.g., how do we route the commodity of interest (e.g. information, energy, traffic etc.) so as to avoid bottlenecks? Is a highly connected network preferable to a sparsely connected system? Where new capacity should be incorporated? In power supply applications, for example,the net work architecture (i.e scale-free or homogeneous [20]) will have a significant effect on system reliability where intermittent equipment failure, demand overload, and flow congestion have to be dealt with to ensure proper system functioning [21], [20], [22].

Algorithms for optimizing flow models in networked systems fall into three main classes: Markov matrix-based [23], linear/mixed integer linear programming methods (ILP) [24],[25] or conductance model analogues, i.e. Kirchoff circuits [26]. These approaches either solve fixed structure networks (in the Kirchoff case) or involve linear models. Our approach by way of contrast allows for both network structure evolution (the so-called rewiring problem [27], [28])as well as non-linear objective functions. To our best knowledge this has not been done thus the approach described represents a significant new direction.

2. The energy distribution network model

We consider an undirected graph $G = (V(G), E(G))$ with vertex set $V(G)$ and edge set $E(G)$. The edge set $E(G)$ of the network can be represented by an adjacency matrix **A** [29] and we will assume that any two edges may be connected with no-self-loops. The resultant flow model has the following canonical form [30]:

$$P = \begin{matrix} \\ TR. \\ ABS. \end{matrix} \begin{matrix} \begin{matrix} TR. & ABS. \end{matrix} \\ \begin{pmatrix} \mathbf{Q} & \mathbf{R} \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \end{matrix}$$

Here **I** is a K by K identity matrix, **0** is a K by N zero matrix, **R** is a nonzero N by K absorbing state matrix, and **Q** is the N by N transition probability (Markov) matrix which directs flows along the network from node i to node j according to the prob- ability associated with the element q_{ij}. For an absorbing network where there is energy consumption at each node the sum of the matrix row elements $q_{ij}$are less than one along each row [30]. If the absorption probability at node i is $q_a i$ then1 $-\Sigma_i^N q_{ij} = q_a$

If the required total power availability at each point in the network is denoted by the **row vector u** and the external power/backup availability at each node is denoted by the **row vector s** then (assuming no losses) we can write the following energy flow conservation equation relating these quantities:

$$\mathbf{u} = \mathbf{uQ} + \mathbf{s} \qquad (2)$$

We observe that the row vector **u** has individual elements $u_i$representing the power flow into each node i. Some of this power is transmitted elsewhere in the network while a fraction is consumed locally.
If we define a matrix $\mathbf{U}_{n\times n}$which has the elements of vector u arranged along its main diagonals follows,

$$\mathbf{U}_{n\times n} = \begin{pmatrix} u_1 & & 0 \\ & u_2 & \\ 0 & & \ddots \quad \cdots \\ & \cdots & u_n \end{pmatrix} \qquad (3)$$

we can define a matrix **D** that represents the power flow between nodes.

Each element d_{ij} of **D** is defined as follows,

$$d_{ij} = u_i \times q_{ij} \tag{4}$$

where q_{ij} is the ij element of **Q**.

$$\begin{pmatrix} u_1 & & & 0 \\ & u_2 & & \\ & & \ddots & \vdots \\ 0 & & \cdots & u_n \end{pmatrix} \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{21} & \cdots & q_{2n} \\ & & \ddots & \vdots \\ q_{n1} & q_{n2} & \cdots & q_{nn} \end{pmatrix} = \begin{pmatrix} u_1q_{11} & u_1q_{12} & \cdots & u_1q_{1n} \\ u_2q_{21} & u_2q_{21} & \cdots & u_2q_{2n} \\ & & \ddots & \vdots \\ u_nq_{n1} & u_nq_{n2} & \cdots & u_nq_{nn} \end{pmatrix} \tag{5}$$

If we define the demand goal at each node as a_i then the 'residual' energy at each node is given by $\delta_i = c_i - a_i$ which can be either Σpositive oΣr negative. If the system is capable of Satisfying its energy needs internally

$$c_i = (1 - \sum_{j}^{N} q_{i,j})u_i$$

that some points are under-supplied and vice-versa. Indeed the network design problem is to mitigate these potential imbalances.

2.1. Network structure evolution

We now introduce the concept of the 'network flow roughness' function $w(N)$ inspired by the use of an analogous property in systems involving stochastic deposition processes [31]. In that particular context it represents the overall uniformity of a thin-film surface being randomly deposited on a substrate. Here it is used, however, as a measure of the overall departure of the network's residual energy 'surface' from to what is manifestly a stochastic flow re-distribution process. We define the network roughness function $w_k(N)$ at the k^{th} iteration as follows:

$$w_k(N) = 1/N \sum_{i=1}^{N} \delta_i^2$$

The function $w_k(N)$ is a quantitative measure of the overall satisfaction of energy demand *throughout* the system. We use it to decide whether or not to accept each flow redistribution step, and simultaneously re-wire the network, using Monte Carlo sampling, in a manner analogous to the Metropolis method [32–34].

3. Results and discussion

We ran the algorithm with the network structure shown in Figure 1 which was generated as a random graph with energy supply and demand values at each point generated from a uniform random distribution.

The improved supply-demand equilib- rium throughout the network is demon- strated in Figure 2 with the decrease in the network roughness function, which is a quantitative measure of the extent of supply- demand satisfaction, quite evident. These results were done at various values of the annealing factor β which controls the rate at which the algorithm progresses towards the optimal structure. These results which have been observed in other cases [34] show that it (the annealing factor) appears to not have a significant influence on the calculations. However, this may be a problem dependent issue which can be investigated numerically by repetitive running of the program

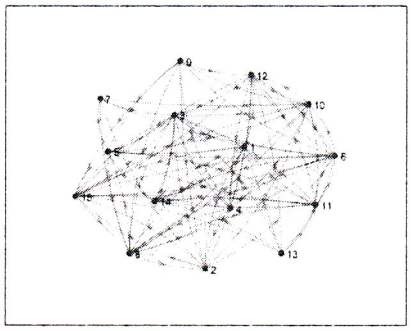

Figure 1: Initial Network Structure

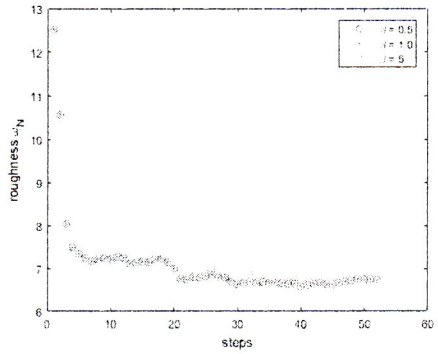

Figure 2: Network Roughness Function with Structure Evolution

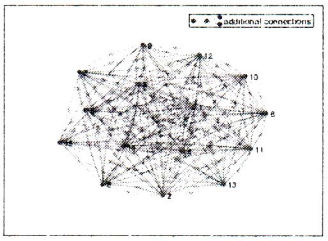

Figure 3: Final Network Structure

The final network structure is shown in Figure 3 with the added links shown in red. This brief numerical example illustrates the capability of the algorithm in evolving novel network structures that better satisfy supply-demand equilibrium throughout the system. In a forthcoming publication [34] we provide further details of the method and more extensive testing of it. The final net- work structure is shown in Figure 3 with the added links shown in red. This brief numerical example illustrates the capability of the algorithm in evolving novel network structures that better satisfy supply-demand equilib- rium throughout the system. In a forthcoming publication [34] we provide further details of the method and more extensive testing of it

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